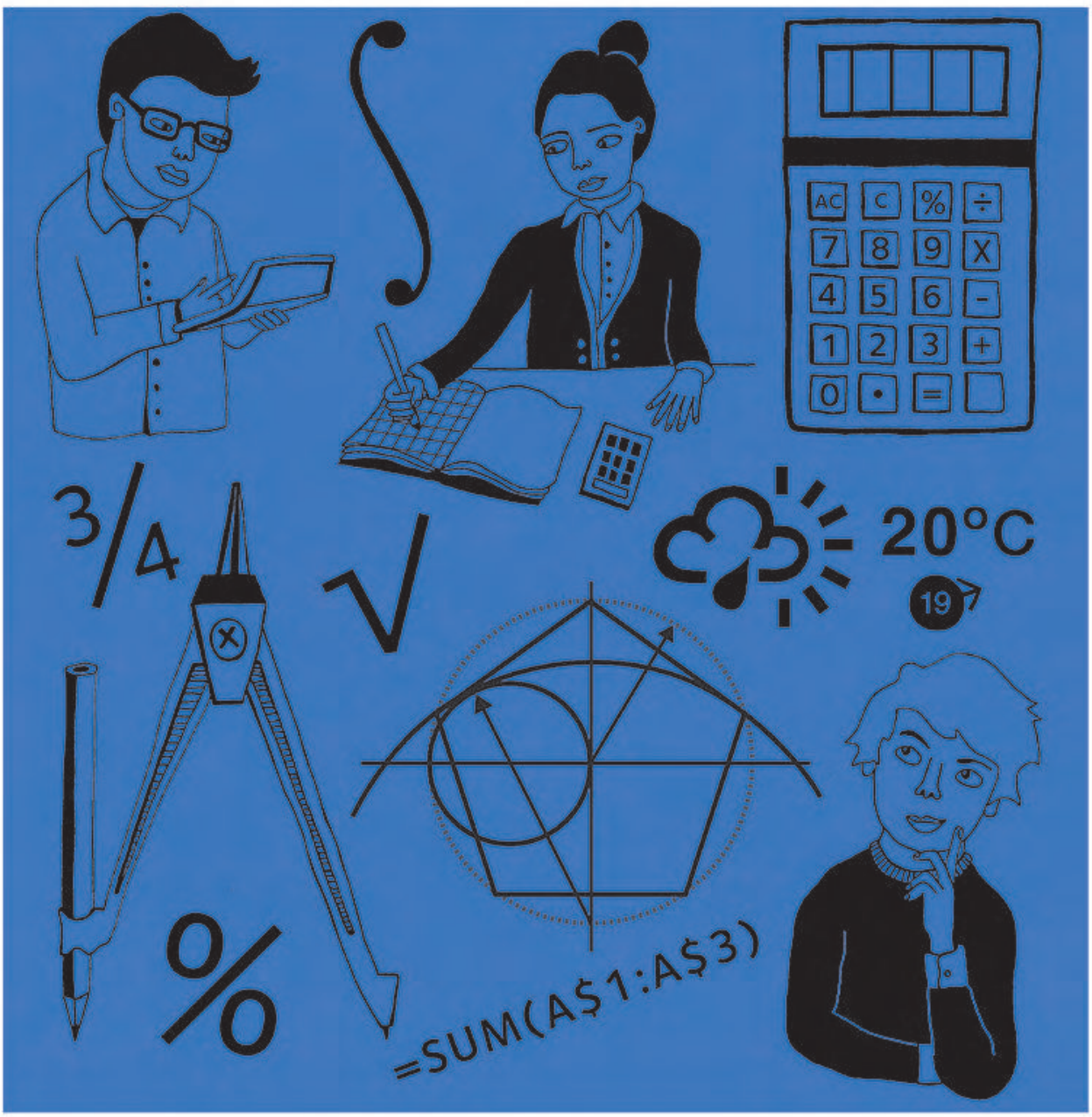
Learning to Teach Mathematics in the Secondary School

A companion to school experience

Edited by **Sue Johnston-Wilder, Clare Lee** and **David Pimm**

4TH EDITION



LEARNING TO TEACH MATHEMATICS IN THE SECONDARY SCHOOL

Learning to Teach Mathematics in the Secondary School combines theory and practice to present a broad introduction to the opportunities and challenges of teaching mathematics in the secondary school classroom. This fourth edition has been fully updated to reflect the latest changes to the curriculum and research in the field, taking into account key developments in teacher training and education, including examinations and assessment.

Written specifically with the new and student teacher in mind, the book covers a wide range of issues related to the teaching of mathematics, such as:

- Why we teach mathematics
- The place of mathematics in the National Curriculum
- Planning, teaching and assessing for mathematics learning
- How to communicate mathematically
- Using digital technology to advance mathematical learning
- Working with students with special educational needs
- Post-16 teaching
- The importance of professional development
- The affective dimension when learning mathematics, including motivation, confidence and resilience

Already a major text for many university teaching courses, this revised edition features a glossary of useful terms and carefully designed tasks to prompt critical reflection and support thinking and writing up to Master's level. Issues of professional development are also examined, as well as a range of teaching approaches and styles from whole-class strategies to personalised learning, helping you to make the most of school experience, during your training and beyond.

Designed for use as a core textbook, *Learning to Teach Mathematics in the Secondary School* provides essential guidance and advice for all those who aspire to be effective mathematics teachers.

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LEARNING TO TEACH SUBJECTS IN THE SECONDARY SCHOOL SERIES

Series Editors: Susan Capel and Marilyn Leask

Designed for all students learning to teach in secondary schools, including those on school-based initial teacher education programmes, the books in this series complement *Learning to Teach in the Secondary School* and its companion, *Starting to Teach in the Secondary School*. Each book in the series applies underpinning theory and evidence to address practical issues to support student teachers in school and in higher education institutions in learning how to teach a particular subject.

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LEARNING TO TEACH MATHEMATICS IN THE SECONDARY SCHOOL

A companion to school experience

Fourth edition

Edited by Sue Johnston-Wilder,
Clare Lee and David Pimm

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This book is dedicated to our children and grandchildren, the maths students of the next generations:

Robin, Beth, Jaclyn, Daniel, Alex, Alastair, Olivia, Rhiannon, Steve, Kai and Daniel

and is offered in memory of Jill Bruce, Rita Nolder, Christine Shiu and Gill Hatch – inspirational teacher-researchers who died before their time.

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Steven Watson is a lecturer in mathematics education at the University of Cambridge. He is responsible for the Secondary Mathematics PGCE course and the Mathematics Education Practitioner Professional Development courses. He teaches on the MPhil/MEd in Mathematics Education and supervises doctoral students. His research is concerned with mathematics teachers' professional learning, both in initial teacher education and for practising teachers. He is currently leading a component of the Cambridge Mathematics Education Project (CMEP), looking at classroom practice and pedagogy at A-level. Steve completed a PhD in Mathematics Education looking at mathematics teachers' professional development at the Shell Centre, University of Nottingham, before taking up his post in Cambridge. Previously, he was a secondary mathematics teacher and latterly head of maths in secondary schools in North-East Lincolnshire.

FOREWORD

There are likely to be few more important jobs in the twenty-first century than that of a mathematics teacher. Knowledge in many of the key contemporary growth areas, and the ability to harness them for the benefit of humanity, requires a competence with the appropriate underlying mathematical ideas. These key areas include:

- the software and hardware associated with information and communication technology;
- finance and economics, including the control of risk;
- design in a variety of fields, from machinery to fabrics, and from graphics to architecture;
- all branches of science, from the biotechnology of genetic engineering and medicine to the physics of cosmology;
- the quantification of performance, and the statistics and modelling involved in research and development in all fields.

For children to be able to hold their own in this century, it is thus critical that they have an appreciation of, a competence in manipulating and a positive attitude towards the big mathematical ideas that are central to the functioning of our global culture. This means understanding the development of these ideas by individuals and groups within particular social contexts, enjoying the elegance and beauty of their patterns and symmetries for their own sake, as well as acquiring an ability to participate in the implementation and the shaping of future advances which rely upon them.

Mathematics teachers, therefore, have a significant part to play in ensuring that the next generation is both excited and well equipped. The job is uniquely challenging in requiring familiarity and engagement simultaneously with the unsullied abstraction of mathematics and the sometimes crude realities of classes of teenagers. However, many of us who have experienced great enjoyment in the role of mathematics teacher have found that the successful bringing together of these diverse worlds is at the same time uniquely rewarding and very worthwhile.

I am pleased and honoured to have been asked to contribute the Foreword for the fourth edition of this book. The previous editions have proved to be very successful and the new chapter is very welcome as it deals with affective aspects of mathematics education, which have recently become even more important. This book has been written by a group of people all of whom I respect as excellent and knowledgeable teachers, both of secondary students and of intending and serving mathematics teachers. Indeed, I know most of the authors well as former PGCE students, MA students, colleagues on research and development teams, and fellow members of committees, working groups and conferences.

The various chapters introduce new teachers to the leading edge of theory and research, made meaningful through practical examples, often drawn from personal experience. Because it will cause careful thinking about the what, the why and the how of teaching mathematics, it is a very important book. I believe it will encourage the tradition of excellence in mathematics teaching which both respects and seeks to build connections within and between mathematics and secondary school students.

Margaret Brown
Professor of Mathematics Education,
King's College London.
November, 2015

SERIES EDITORS' PREFACE

The fourth edition of *Learning to Teach Mathematics in the Secondary School* is one of a series of books entitled *Learning to Teach (subject name) in the Secondary School: A Companion to School Experience* covering most subjects in the secondary school curriculum. The subject books support and complement the generic book *Learning to Teach in the Secondary School: A Companion to School Experience*, 7th edition (Capel, Leask and Younie, 2016) which deals with aspects of teaching and learning applicable to all subjects. This series is designed for student teachers on different types of initial teacher education programmes, but is proving equally useful to tutors and mentors in their work with student teachers.

The information in the subject books does not repeat that in *Learning to Teach*, but extends it to address the needs of student teachers learning to teach a specific subject. In each of the subject books, therefore, reference is made to the generic *Learning to Teach* text, where appropriate. It is recommended that you have both books so that you can cross-reference when needed.

The positive feedback on *Learning to Teach*, particularly the way it has supported the learning of student teachers in their development into effective, reflective teachers, encouraged us to retain the main features of that book in the subject series. Thus, the subject books are designed so that elements of appropriate theory introduce each topic or issue, and recent research into teaching and learning is integral to the presentation. In both the generic and the subject books, tasks are provided to help you identify key features of the topic or issue and apply them to your own practice. In addition, the requirement for material to be available to support student teachers' work at Master's level in PGCE courses in England has been met in the latest editions by the inclusion of advice about working at this level. The generic book referred to above also has a companion Reader (*Readings for Learning to Teach in the Secondary School*) containing articles and research papers in education suitable for 'M' level study.

We as editors have been pleased with the reception given to the earlier editions of this book as well as to the *Learning to Teach* series as a whole. Many subject books have moved into their third or fourth editions and others are in preparation. We hope that whatever initial teacher education programme you are following and wherever you may be situated you find the fourth edition of *Learning to Teach Mathematics in the Secondary School* supports your development towards becoming an effective, reflective teacher of mathematics. You should also find the companion practical book, *A Practical Guide to Teaching Mathematics in the Secondary School*, of value. Many of the authors contributing to the Learning to Teach Series are also contributing to the research summaries on www.MESHGuides.org. The MESHGuides build on the subject series and are intended to support you to develop evidence-informed practice throughout your career. Above all, we hope you enjoy teaching mathematics.

Susan Capel and Marilyn Leask
March 2016

PREFACE TO THE FOURTH EDITION

Teachers continue to work in a climate of constant change. You can expect change to be a feature of your teaching career; if anything, the pace of change will increase. In this fourth edition, we continue to reflect the waves of change that have happened since the third edition was published just five years ago.

- There have been recent major changes in the way that teachers are educated in England. Some schools have become training schools, resulting in less HEI involvement in educating teachers. Teach First has expanded, a route which aims to get teachers into the classroom quickly and support their learning as they teach. This has, in some cases, placed more emphasis on the practicalities or the craft of teaching and less on the theory that backs up ways of acting in the classroom.
- The growth of academies and free schools in England has been a significant change, as such schools are regarded as independent, despite receiving state funding. As a result, they are not required to teach the National Curriculum and some academies and free schools are using this freedom in innovative and exciting ways. However, Ofsted will continue to use results from Key Stage 2 national tests and the GCSE examination at age 16 to judge the quality of these schools, which may limit their freedom.
- A new National Curriculum for England was published in 2013 (DfE, 2013). The full curriculum for Key Stages 1–4 will be taught from August 2016, although many schools have been using it to devise their schemes of work from its publication.
- ICT and the use of 'apps' has become more prevalent in schools for use by both teacher and pupils and there are increased opportunities and expectations for wider use. In England, the new National Curriculum for Mathematics (DfE, 2013) does not actively encourage the use of digital technology and has an emphasis on traditional methods of calculation, which may mean that the use of ICT in learning mathematics does not receive the attention it merits in some schools.
- In England, the latest revision of the National Curriculum (DfE, 2013) sets out computing as a discrete programme of study. While this does not mean that computing must be learned in that way, the emphasis on fundamental principles and concepts of computer science will make discrete lessons likely. However, it may be that mathematics could contribute to the 'creative projects' (DfE, 2013, p. 232) that are part of the computing programmes of study.
- Functional Mathematics has been defined and is now part of Key Stage 3, GCSE and post-16 courses in England.
- GCSE and post-16 assessment methods have changed and continue to change.
- *Mathematics: Made to Measure* was published in May 2012. It was based on mathematics inspections carried out between 2008 and 2011. In the foreword to the report, HMCI stated that Ofsted would:

- produce support materials to help schools identify and remedy weaknesses in mathematics
- raise ambition for the mathematics education of all pupils by placing greater emphasis in school inspection on:
  - how effectively schools tackle inconsistency in the quality of mathematics teaching;
  - how well teaching fosters understanding;
  - pupils' skill in solving problems;
  - challenging extensive use of early and repeated entry to GCSE examinations.

(p. 5)

- The Ofsted (2015) handbook refers to progress in mathematics being assessed by drawing on evidence from other subjects in the curriculum:

  Inspectors must evaluate and report on the difference in average point scores at GCSE at the end of Key Stage 4, between disadvantaged pupils and other pupils nationally, and between disadvantaged pupils and other pupils within the school. They must report on the extent to which any such gaps are closing, considering in-school gaps in the context of national gaps.

  (Ofsted, 2015, p. 55)

- Ofsted (2015) also states that in outstanding schools:

  Leaders plan, manage and evaluate study programmes so that learners undertake highly individualised and challenging learning that builds on their prior attainment, meets all the requirements of 16 to 19 provision and prepares them very well for future employment. Learners without GCSE grades A* to C in either English or mathematics follow appropriately tailored courses in English and/or mathematics. The considerable majority make substantial and sustained progress towards grade C or above.

  (Ofsted, 2015, p. 66)

- There is now an increasing emphasis on all students progressing well. The Ofsted inspection handbook asserts that in good schools:

  Pupils' progress is above average or improving across most subject areas. Overall progress of disadvantaged pupils, disabled pupils and those with special educational needs is above average or improving. From different starting points, the proportions of pupils making and exceeding expected progress in English and in mathematics are close to or above national figures. The progress of the very large majority of disadvantaged pupils is similar to or improving in relation to other pupils nationally.

  (Ofsted, 2015, p. 58)

In addition, the standards for qualified teacher status (QTS) have been revised (DfE, 2011a). Courses leading to qualified teacher status may or may not include a PGCE element and most PGCE courses now include Master's-level study. Each chapter in this book is designed

to introduce you, whichever route into teaching you have chosen, to the practicalities of teaching. The chapters also discuss the theoretical background that will allow you to develop a confidence in how to act, by knowing why your actions in the classroom are likely to be successful.

All the chapters in this edition have been revised and a new chapter has been added on the affective dimension in learning mathematics: 'Learning mathematics: an affective focus'. Chapters 1 and 3 have been extensively revised, with new authors for Chapter 3 giving an up-to-date focus on the mathematics that you will teach and how you will help your students learn. The chapter on assessment now covers all aspects of assessment; both formative assessment and how that affects the way that you will teach *and* summative assessment, which sets the context in which you will teach. The tasks in each chapter have once again been revised in order to prompt critical reflection and to support you in developing your thinking and writing to Master's level.

There are other aspects of being a secondary mathematics teacher that do not change over time and so it is right that many core aspects of this book have stayed the same. Here, we take the opportunity to acknowledge and thank authors whose work featured in previous editions – Ruth Edwards, Maria Goulding, Gillian Hatch, Peter Johnston-Wilder, Ann Kitchen, Christine Shiu and John Westwell – as this edition still retains much of their wisdom. The changes made in this edition reflect the changed environment in which you are preparing to work.

INTRODUCTION

Learning to teach mathematics is not something you will do in just a year. This book might have more appropriately been titled '*Beginning* to learn to teach mathematics in a secondary school'. For however long you continue to teach mathematics, you will also continue to learn about the extremely complex and challenging profession which you have decided to join. Having said that, the lessons you learn about teaching mathematics as a student teacher will have a significant impact on your future career. This book will guide you through the very important first stage of your formal development as a mathematics teacher.

Start to think of yourself as a mathematics teacher right from the beginning of your course. The career on which you are embarking is multi-faceted. Not only will you be developing as a teacher, but you will also be developing as an educational researcher, as a mathematician, as a writer, as a counsellor, as a team worker and as a reflective practitioner. You will find yourself stretched intellectually, physically and emotionally; there may be times when you wonder why you ever decided to teach. However, along with the challenge, there will also be the rewards and satisfaction that come from working with young people and with your colleagues.

Making the most of your school experience

There have been some significant changes in the structure of teacher education courses in recent years. As well as there being more central government control over the aims and content of courses, there has also been a considerable increase in the time spent by pupils in schools. Many people see this as a positive development, arguing that it is essential that new teachers learn 'on the job' in the school environment. However, there is a danger that you will not get the maximum benefit from your school experience *unless* you use it as an occasion to reflect and learn from a full range of experiences. Here is a list of ways that you can learn from and through your school experience.

Reflective journal

It is widely accepted that reflecting on your experiences in a structured way is essential if you are to develop as an effective practitioner. You will be bombarded by experiences during your time in school, so it is important that you do indeed make time to reflect. Perhaps the most effective way of supporting this process is by means of writing down your thoughts, as a record that you can look back on. Indeed, the practice of using a reflective journal as a tool for personal development goes back many centuries, and many people now use internet

blogging for this purpose. If you establish the habit of keeping a reflective journal during your training, you will be well prepared to continue the practice throughout your career.

Classroom research

During your training, you will spend much of your time in classrooms. For a significant proportion of this time, you will not be primarily responsible for the teaching. Instead, although you may be supporting a teacher, you will have an excellent opportunity to observe and investigate aspects of classroom life in some detail. While student teachers often complain that they have to spend too long just observing, many experienced teachers complain that they have too few chances to observe their colleagues or their pupils. Using this special time effectively to undertake a variety of focused classroom research tasks will add significantly to your knowledge and understanding of teaching and learning.

Investigating the school and department

Given how much time you spend in school, it is important that you are familiar with how both the school and its mathematics department are organised and operate. However, you should go further than just finding out enough to survive. You have the opportunity to develop an understanding of factors that lead to an effective school and department. Comparing your school and department with those of fellow student teachers will also support this process. The teachers in your school will also benefit from the presence of a student teacher who can, on occasion, ask perceptive questions and cause them to reflect on their policies and practices.

Researching the curriculum and resources

Even if you have gone straight from school to university to teacher education, you can still expect the mathematics curriculum to have changed in the intervening years. It is important to understand the structure and content of the curriculum for different phases of schooling and to be familiar with associated qualifications and/or assessment arrangements. There is an abundance of teaching resources available to mathematics teachers. Your period of education is an ideal time to investigate these resources and to evaluate them critically. You may find that your department will also appreciate hearing about the results of such research.

Studying mathematics

As you begin to teach, it is important that you continue to study mathematics. This can include both exploring new areas of mathematics and going deeper into areas you have previously studied. In particular, you need to develop your subject knowledge in those areas of the mathematics curriculum about which you are less confident. However, even with topics with which you have no difficulty, you can enhance your teaching by thinking more deeply about the concepts and connections related to each topic. A further aspect of studying the subject involves learning more about its diverse applications and history.

Interviewing staff and students

You will have plenty of opportunity to discuss education informally with both staff and pupils, but it is also valuable to explore issues more formally. At the end of lessons, teachers may well need to prepare for the following lesson or they may want a break. However, arranging to interview staff, even for a short time, will mean that both you and the teacher have a chance to reflect in more depth about her or his thinking and practice and to relate it to your own developing practice. Interviewing students in a small group or perhaps in a pair within the wider school environment would allow you to listen to what they really think about some aspect of their education.

Reading about education

With much of your time spent in schools during your course, it is important that you make time to read widely about education. This will contribute towards the development of your personal theoretical base from which you can reflect more effectively on your school experience. It is worth planning to include reading time and visits to your education library during your school experience.

Teaching

Many student teachers consider that taking sole responsibility for teaching mathematics to a class is the main point of their school experience. Obviously, you have to have enough opportunity to learn how to cope with the complex demands that teaching lessons on your own can bring. However, in order to develop your understanding of and skill at teaching, you will need to have experiences in which you just focus on particular aspects of teaching or learning. You can do this by teaching just a small group, or a pair or an individual, by taking responsibility for just part of a lesson or by team-teaching with a colleague. Whatever form the teaching takes, be sure that it also includes opportunities for planning, assessment and evaluation.

How to use this book

The book is divided into fourteen chapters, each of which addresses an important theme. Every chapter has an introduction and objectives, which are designed to clarify the key ideas addressed in the chapter. The summary at the end of each chapter highlights the main points made. You may find it helpful to read a chapter prior to addressing the theme in a tutorial (either at school or university). You may also find it helpful to dip in and out of the book when you want to read something to support your reflection on particular school experiences. Obviously, you will need to read more than just this book, so, in order to support your further reading, each chapter ends with some recommendations for relevant reading on the same theme. In addition, the chapters are fully referenced, thereby offering you alternative additional sources of further reading.

An important feature of this book is the tasks, which comprise a key element in each chapter and provide you with many suggestions for making the most of your school experience. There are tasks to support each of the types of learning experience addressed in the relevant

section and to prompt you to reflect on and to evaluate your learning in school. Your course will place various requirements upon you, but there will be opportunities to negotiate with your school mentor the details of how you use your time in school. You may wish to suggest to your mentor some of the tasks from this book as possible elements of your school experience programme. The tasks will be of more value if you can talk them through with your mentors or your fellow student teachers.

At the end of the book, you will find a glossary of useful terms (Appendix 1) and a collection of useful addresses for resources and organisations (Appendix 2). You will likely add to these as you progress through your career.

The course will be hard work; at the end, you will emerge as a qualified 'beginning' teacher of mathematics, prepared to inspire and encourage the next generation of young people. You are much needed. Welcome to the challenge.

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1 Why do we teach mathematics?

Robert Ward-Penny

Introduction

One of the most common questions that you are bound to hear as a secondary mathematics teacher is 'Why do we need to do this?' Although your students may sometimes ask you this just to distract you from your work, or to stop them from having to do theirs, they are touching on some important and far-reaching concerns. What makes it worthwhile learning a particular topic in mathematics? Why are students in secondary schools expected to spend so much time and effort on this one curriculum area? What is the point of teaching and learning mathematics?

Questions about the purpose of education rarely have simple answers; mathematics teaching is an endeavour with many aims and an enterprise with many stakeholders. However, these questions are important ones that will underpin all the work you will do as a secondary teacher. Reading through this introductory chapter, and working on the tasks, will help you explore some of the fundamental ideas that different people, including you, have about mathematics education. Your understanding of these ideas will help you answer the 'Why do we need to do this?' question, both for yourself and for your students, and also support you in making informed practical teaching choices.

Objectives

By the end of this chapter, you should:

- understand better how your own experiences have already shaped your views about mathematics education;
- be aware of some of the different reasons why mathematics is taught in schools, and why it is considered to be so important;
- be able to make connections among the different philosophical purposes of mathematics education and various practical aspects of teaching and learning mathematics;
- know some of the organisations that influence practice in mathematics education;
- be able to articulate more clearly your own current rationale for mathematics education.

Mathematics and you

You are about to begin exploring the field of mathematics education, but in reality you are not a beginner; you already have a wealth of experience upon which to draw. Whether you have studied mathematics at degree level or not, you will have spent many hours of your life engaged in learning and using mathematics – and in being taught it. Your views about what mathematics is and what mathematics teaching looks like, and your own philosophy of mathematics education, have therefore already been informed by all the memories, feelings and opinions you have gathered as a learner. It is therefore a valuable exercise to begin by looking back at and reflecting on where you have come from, what you are already aware of, and what opinions and biases you might bring to your training. Task 1.1 leads you through this process.

Task 1.1 Mathematics – your story so far

This task is structured to help you consider your experiences of mathematics and mathematics education to date. Give yourself some undisturbed time to read each of the bulleted points below slowly and reflect upon the thoughts stimulated by the questions. Note down any ideas or memories as they occur in a reflective journal.

- What is your earliest memory of learning mathematics? Call to mind where you are, what you are doing and how old you are. Is it a positive memory? What is your most recent memory of doing mathematics? Think about why you were doing it and how you felt at the time.
- How do you rate your intellectual capability in mathematics? Do you believe that your ability is something that you were born with or is it something that has developed over time? How does your mathematical capability compare with that of others? What basis are you using for your comparison?
- Who do you think was your best mathematics teacher? Consider what it was about this person that impressed you. Try to remember some particular moments that exemplify all that was good about their teaching. What aspects of this person's teaching would you like to emulate?
- Who do you feel was your worst mathematics teacher? Think what it was about this person that led you to such a judgement. Try to remember some specific occasions that exemplify what was poor about their teaching. Which aspect of this person's teaching would you most wish to avoid replicating your own practice? Is this likely to be an issue for you?
- Try to remember some occasions when learning mathematics was difficult and some when it was easy. Why was there a difference? Consider other people with whom you have learnt mathematics. Did they find it easier or harder than you? Why do you think that was? Do you think you have a preferred way of learning mathematics?
- Think about your friends or peers from secondary school who were taught in different groups or sets for mathematics. In what ways do you think their experiences of learning mathematics would have been qualitatively different from your own?

(continued)

Task 1.1 *(continued)*

- Consider each of the different stages of your own mathematics education. At each stage, what did you think was the point of learning mathematics? Did you ever discuss this with a teacher, advisor or lecturer? When and why did you decide to become a mathematics teacher?

Having looked back over some of your past encounters with mathematics, you are now in a better position to look forward. In order to grow and develop as a mathematics teacher, you will need to go on reflecting about mathematics, about education and about your place in relation to both. You may like to return to your notes during or after your teaching practice, to help place your own experience in a wider context.

Key purposes of mathematics education

It might seem peculiar to begin this book with a chapter that focuses on abstract issues and the purposes of mathematics education; you may be keen to get on with the likely somewhat daunting matter of surviving in a classroom full of students. However, it is important to understand at this early stage how widely such issues can impinge upon practical matters. You, your students, your school and your government will all have different aspirations and ambitions related to what happens in your classroom. (And remember your classroom is not solely a mathematics one: see Capel, Leask and Turner, 2013, Unit 7.2.) It is important that you are aware of these from the very outset, so that you can better balance the various needs, outcomes and pressures. This section will therefore outline six of the purposes of mathematics education and discuss how each might be reflected in your practice.

Task 1.2 Painting by numbers

To begin with, consider this somewhat mundane mathematics question:

> Caroline has used three cans of paint to cover one room. How many cans of paint will she need to cover four rooms? Write down a formula to describe this situation.

Imagine that one of your students has asked you 'Why do we need to do this?' Before reading on, consider how you might answer their question. How many different responses could you offer this student?

Everyday mathematics and the development of numeracy

Perhaps the most obvious answer to this student's question is 'In order to check you can multiply three by four, and in case you ever need to find out how many cans you should buy to paint four rooms'. Although this scenario may be unlikely in isolation, there is a place in the secondary classroom for the practice of simple problems with immediate applications. One of the most fundamental purposes of mathematics education is to ensure that all learners

can apply basic techniques of number and measure in commonplace situations. This area of mathematics is sometimes called *numeracy* or *functional mathematics*.

Part of this involves an awareness of certain basic aspects of modelling and some specific assumptions that are tacitly involved in this mathematical question to render it answerable, including whether the other rooms are all the same size as the first. Even if the dimensions are comparable, it is not a question of volume but whether there is the same surface area to be painted in each room (see Keitel, 1989, for an account of a related painting task in the classroom, the discussion of which does not go at all as the author of the article expected).

As a successful learner of mathematics, you might be surprised at some of the basic techniques that secondary mathematics teachers are often required to teach. The remit of the mathematics department not only includes teaching all students to use units of measurement and perform simple arithmetic, but also checking that each learner can read a clock and use money confidently. You might also be shocked to find out that many who leave school still struggle with applying mathematics in everyday contexts: one meta-analysis of 13–19-year-olds conducted by Rashid and Brooks (2010) estimated that 22% of young people in England are not able 'to deal confidently with many of the mathematical challenges of contemporary life' (p. 71). Many learners subsequently choose to enrol on adult education numeracy courses in an attempt to gain such basic competence that they missed out on at secondary school, and in addition to overcome the difficulties of getting a job, which can be exacerbated or even caused by poor numeracy. There is certainly still a need for mathematics teachers to ensure that all their students can use basic mathematical ideas and techniques confidently and effectively.

Straightforward arithmetic word problems such as Caroline's paint problem place a mathematical operation in a context and may start to help students make connections between mathematics and the outside world. However, there are many other tasks that foreground everyday abilities much more effectively and start to bring reality (in all its various forms and guises) into the classroom. For instance, you might ask your students to budget for a family holiday using brochures; to plan a trip to the seaside using timetables; to compare different mobile phone tariffs in a catalogue; to read and interpret some utility bills. These tasks may not always be at an appropriate level of difficulty though – and as they become more involved and realistic, they can also become more time-consuming, lasting for entire lessons or even longer. One of your responsibilities as a mathematics teacher will be to determine if, when and to what extent such tasks and explorations are appropriate, as well as to balance the time you spend working towards each of your goals.

Preparation for work and vocational development

A second answer to your student's question is 'Because resolving this problem practises certain mathematical ideas and techniques that you might need to use in your job in the future'. For example, in this instance, the relationship between the number of rooms R and the number of paint cans C can be written as $C = 3R$. While this particular formula is of limited use, many professions involve formulae of some sort: nurses use them to calculate safe dosages; account managers use them when setting up spreadsheets; special effects organisers can use them to calculate safe distances when working with pyrotechnics. Therefore it is valuable for students to practise coming up with, writing and reading formulae, since the ability to

express relationships symbolically and to work with algebraic expressions is essential in so many careers involving science, technology or engineering. Similar arguments can be made for much of the mathematics curriculum; for example, probability is used by insurance companies, weather forecasters and call centres, whereas statistics is deployed in fields ranging from art history to zoology.

While some aspect of mathematics can be used valuably in almost any career, there is also at present a focus on jobs that require a significant amount of mathematical competence. Official reports – such as Roberts (2002) – recognise that there is a shortfall of individuals in STEM (science, technology, engineering and mathematics) careers; although such jobs are essential in a modern economy, there are not enough graduates in many STEM fields, and there is a deficit in some significant areas such as finance. Consequently, there are many interest groups who work to promote STEM subjects and careers, and you may come across some of these either indirectly or directly in your work as a mathematics teacher.

It can be argued that repetition and practice are effective in preparing students for employment, as they work towards ensuring that learners can perform a set range of relevant tasks correctly and reliably. There are many other approaches to mathematics teaching which foreground the goal of preparing students for future occupations, combining the 'functional mathematics' described above with concepts and methods borrowed from outside mathematics. Taking examples from within STEM fields, you might: show your students how to work with numbers in standard form by borrowing figures and contexts from astronomy; illustrate sample spaces in probability by talking about dominant and recessive genes; practise algebraic substitution with authentic formulae taken from a field such as engineering. Such exercises can be very valuable, as they can demonstrate to your students how mathematics is used via genuine contexts, and that it is not just limited to contrived and simplistic scenarios (such as Caroline's painting problem). You may even choose as a mathematics teacher to develop this approach further, using fuller and longer cross-curricular projects in your classroom, perhaps bringing in any expertise that you have from a previous career in industry or another discipline, or working with members of staff from another department.

The aim of preparing students to use mathematics in occupations outside of school can also influence how you use technology in the classroom. Adults in employment have access to a range of technologies, including calculators, spreadsheets, dynamic geometry software (such as *Cabri* or *The Geometer's Sketchpad*) and even computer algebra systems. You will therefore need to consider when, and in what ways, you allow your students to use these tools in their work, and how often you insist that they work unaided instead. Similarly, workplaces often involve people working in teams: is this something that you feel should be replicated more in the classroom?

Thinking techniques – habits of mind and personal development

> Thinking mathematically is not an end in itself. It is a process by which we increase our understanding of the world and extend our choices. Because it is a way of proceeding, it has widespread application, not only to attacking problems which are mathematical or scientific, but more generally.
>
> (Mason, Burton and Stacey, 1982, p. 178)